WHY AM I DOING THIS? AN EXPLORATORY STUDY OF WHETHER STUDENTS' CONCEPTUAL KNOWLEDGE INFORMS THEIR PROCEDURAL KNOWLEDGE IN MATHEMATICS.

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Abstract

The main goal for teaching and learning mathematics is to ensure understanding of concepts and procedures aimed at solving tasks. The relationship between conceptual and procedural knowledge in the study of mathematics is of interest to students and teachers, particularly on whether the sequencing of the two knowledge types affects learning. This study aims to highlight the experiences of international under-graduate students of Kenyan origin in the United States with both conceptual and procedural knowledge. It also examines the impact of writing on acquisition and retention of mathematics ideas. Results showed that students prefer conceptual-to-procedural sequencing, although procedural-to-conceptual learning is beneficial in some cases. The findings are analyzed and discussed based on previous research on the two knowledge types.

Keywords: **Conceptual knowledge, Procedural knowledge, Mathematics**

Introduction and Statement of Problem

The study of mathematics involves the use of many formulas and steps to complete tasks (Hiebert, 2013). To correctly solve questions and tasks, a student should learn and possess mathematical knowledge. For student mathematical learning to occur, they should experience both conceptual and procedural knowledge (Rittle-Johnson, Alibali, Haverty, Heffernan, Koedinger & Nhouy vanis-vong, 1999). According to Rittle-Johnson, Schneider and Star, (2015), these two knowledge types build on each other. However, some students may follow procedures correctly, get the correct solution, yet be unable to explain why those steps are correct (Hiebert, 2013), an aspect portraying lack of conceptual knowledge, making them question, "Why am I doing this?" This raises the question of whether there is a relationship between procedural and conceptual knowledge (Rittle-Johnson, Schneider & Star, 2015) and consequently, if learning of the two knowledge types is bidirectional or unidirectional. Rittle-Johnson et.al., (1999) contend that there are only a few studies that have directly examined relations between the two knowledge types.

The procedures followed in solving mathematics tasks involve writing and calculations (Porter and Masingila, 2000). Through writing, students' conceptual and procedural competence and retention is enhanced (Adu-Gyamfi, Bosse and Faulconer, 2010). Through writing, also, students and teachers assess the mathematical understanding of learners. This study seeks to highlight the experiences of undergraduate mathematics students from Kenya with the two knowledge types, their preferred instructional processes, and the impact of writing on their learning, with an aim of exploring whether their learning is unidirectional or bidirectional.

Research Questions

In my study, I sought to answer the following research questions:

a) What are some of the procedures that mathematics students carry out or have carried out when solving mathematics tasks?

b) Are these procedures informed by the students' conceptual knowledge?

c) What perceptions do these students have on the teaching of procedural and conceptual knowledge?

Literature Review

Student learning inculcates attainment of fundamental concepts and correct procedures for solving problems (Rittle-Johnson et.al., 1999). Indeed, mathematical competence rests on development of learners' conceptual and procedural knowledge. Conceptual knowledge is the cognition of abstract and general concepts, operations and relations (Star, 2005) while procedural knowledge is the mastery of procedures. Although the two knowledge types are related (Star, 2005), the developmental relations between them are not well understood (Rittle-Johnson et.al., 1999). There has been a long-standing debate on whether the acquisition of these knowledge types is unidirectional or bidirectional (Rittle-Johnson, Schneider& Star, 2015). Unidirectional relationship asserts that only acquisition of conceptual understanding leads to development of procedural knowledge and not vice-versa (Broody, 2007) while in bidirectional learning, conceptual and procedural knowledge develop iteratively, with gains in one leading to gains in the other (Rittle-Johnson et.al., 1999).

To gauge student learning, assessment is inevitable (Adu-Gyamfi, Bosse, & Faulconer, 2010). Classroom assessment should reflect the mathematics that students should know and be able to do (National Council of Teachers of Mathematics (NCTM), 1995). One way of assessing students' learning is having them solve tasks through writing and/or computations (Adu-Gyamfi, Bosse, & Faulconer, 2010). Moreover, Star (2007) contends that conceptual and procedural knowledge types should be assessed independently, to study the relations between them. He recommends the use of multiple measures for each knowledge type. Schneider and Stern (2010) assert that conceptual knowledge could be measured using various tasks that evaluate the correctness of examples and/or procedures and may demand provision of definitions and explanation of concepts. These tasks should be relatively unfamiliar to participants to ensure that participants derive answers from their conceptual knowledge, rather than implementing known procedures for solving the task. Procedural knowledge on the other hand could be assessed through problem solving, to measure the accuracy of answers or procedures. Procedural tasks are familiar and mostly involve problems that people have solved before, and thus should know the steps.

Methods

The study comprised an in-depth, semi-structured interview protocol with several open-ended questions as recommended by Fontana and Frey (2005). I conducted three interviews to elicit viewpoints of the respondents' experiences with the two knowledge types.

I used the multiple data sources for purposes of triangulation to ensure reliability and validity of the study as recommended by Denzin and Lincoln (2005) and to ensure a deep understanding of the phenomenon under investigation as well as to help in overcoming the biases emanating from one data source (Creswell, 2009).

The participants were all undergraduate students. There was one junior male student, Carl, and two sophomore females, Pamela and Sara. (all names are pseudonyms). They were chosen based on criterion sampling (Xie & Sharma, 2005), the criteria being international students who have studied both in Kenya and in the United states (US) and having taken at least one undergraduate math course.

Researcher Role

Before starting my graduate studies, I was a high school mathematics teacher in Kenya for one year. Here, I involved students in explaining their solution paths to small groups and before the whole class. Although some could explain boldly and correctly, others did not have a strong grasp of their concepts. This disposition has greatly influenced my choice of research topic and development of research questions.

As part of my graduate studies, I have taken a course on Internship in Mathematics Education where a large part of my assignment involved following a mathematics instructor then writing reflections on how students' best learn and the role of a teacher in student learning. I am also a teaching assistant and have taught a mathematics course called teaching mathematics via problem solving where acquisition of conceptual and procedural knowledge is emphasized. The students are provided opportunities to productively struggle with mathematics questions then fully explain their solution paths instead of just giving the final solution. I am aware that this disposition places me in an insider position and could influence my interview questions and reporting since I may create a bias towards my experiences.

Findings

Two themes emerged from the data findings as follows: a) unidirectional versus bidirectional learning, b) writing and assessment

Unidirectional versus bidirectional Learning*:* The participants reported that their knowledge acquisition is both unidirectional and bidirectional. On conceptual-to-procedural learning, the participants felt that in most of their learning, conceptual knowledge led to procedural fluency. They added that mathematics makes sense to them when concepts are first explained followed by homework questions requiring application of the taught concepts. As Sara (not her real name) said, "I learn best when the teacher introduces theorems and then gives us home works to do in the same class." She added that most of her elementary school mathematics made sense because she did guided learning; the teacher taught concepts then guided them in applying the concepts to solve assigned homework questions. Pamela appreciated guided learning as well. She mentioned that, "I liked what my high school teachers did. They would do one-on-one teaching where they would explain to individual students why some concepts were the way they were."On the same note, Carl articulated that:

> I tend to believe that the best way to teach is the teacher trying to put the concept forth and then trying to see if you understood. And how do you see that we have understood it? Maybe give us one or two questions and ask us, can you work on these within the lecture? That will be amazing.

Clearly, the students felt that procedures become more meaningful when they have already grasped the embedded concepts.

Concerning procedural-to-conceptual instruction, participants reported to have had experiences in their learning where teachers taught procedures before explaining the underlying concepts. They however felt that this would sometime lead to confusion and memorization of procedures without understanding. Pamela, for instance, said that she was taught on cross cancellation of fractions at class five. She added that, "It made me get confused and I went as far as form 1 without fully grasping it…" She added that, "For some time, I did not understand what was a numerator, denominator or reciprocal. I think it was harder because the terms were tough and did not make sense to me although I could use them in computations." To solve a question on division of fractions, Pamela used the procedure of multiplying the numerator of one fraction by the denominator of the second fraction, i.e., $\frac{3}{8} \div \frac{1}{2} = \frac{3\times2}{8\times1} = \frac{3\times1}{4\times1} = \frac{3}{4}$.

When given the same question, Carl and Sara multiplied the first fraction by the reciprocal of the second fraction, i.e., $\frac{3}{8} \div \frac{1}{2} = \frac{3}{8} \times \frac{2}{1} = \frac{6}{8} = \frac{3}{4}$. When asked why they approached the question this way, Carl said, “My teacher in class six said that whenever you are dividing two fractions, you must flip the second fraction then multiply but I do not know why.” Sara, on the other hand said, “I know it is correct but I cannot explain… But at least I can write it so I am good.” Both Carl and Sara encountered and memorized procedures before learning the concepts and although they could do the computations, they could not explain why they were correct. This resonates with Hiebert’s (2013) sentiments that getting the final answer correct does not necessarily depict a deep understanding of the embedded concepts.

In some cases, however, the respondents reported that grasping of procedures made learning of underlying concepts easier since they could relate more easily. For instance, Carl said that he was taught about dividing fractions by multiplying the first fraction by the reciprocal of the second fraction at class six. At class eight, he was then taught on the concept of conjugation. For instance, the conjugate of division is multiplication and the conjugate of a number is its reciprocal. According to him, conjugation made more sense since he had been using it for some time although he did not know the name for the concept. He added that, “I like that I was taught multiplication before its conjugate which is division because you cannot learn division without knowing multiplication.”

In general, the respondents reported that acquisition of conceptual knowledge automatically led to acquisition of procedural knowledge. However, when procedural fluency is first emphasized, it may or may not lead to learning of conceptual knowledge. It hence came out that the learning of these two knowledge types is unidirectional for most parts and bidirectional in some cases.

Writing and assessment: The second theme is on students' experiences with writing as part of assessment. Participants felt that writing is important in their mathematics learning since it enhanced retention. The commonly used modes of assessment included use of worksheets, tests and verbal questions. All participants noted that most of their learning has been marked with the use of tests and worksheets. Carl reported that, "Most of the times in my undergraduate classes, my teachers introduce a concept, explain it, then administer a worksheet containing questions for us to attempt…I like this since I can tell if I understood." This idea was reinforced by Sara who also liked the idea of using worksheets as she said, "For me, I have to do computations to understand ideas. When the teacher explains, I only get the rough idea but when I do it later using the worksheets and past test papers, I understand better." Pamela also added that, "Math can be complicated, so I want to relate to what is being taught. To do this, I take notes when the teacher is teaching then I attempt related questions later."

About use of verbal questioning to check their understanding, Pamela said that some of her teachers would let students do questions under their supervision and "If you write a wrong answer, the teacher would ask you on the spot why the answer was that way and let you explain it before telling you how it should be done." Sara added that her high school teachers would put them in groups, assign them questions, then at times have them explain their reasoning on the board to the whole class. If the explanation was vague or wrong, someone else in the classroom would correct it and explain how it should have been done. She also mentioned that, "I learn more when I work out the questions myself instead of copying what the teacher writes."

All participants appreciated the use of tests as a way of assessment. They felt that by working on questions related to the taught concept, it helps them to test how well they have understood the concepts and help them to know what they need to read more or seek assistance on. Moreover, they felt that their learning of math has been the same both in Kenya and in the USA.

In summary, teaching of these two knowledge types must be intertwined with assessment for it to be efficient. From the findings, the two knowledge types should be tested simultaneously as both need to be developed as recommended by Rittle-Johnson et.al., (1999).

Conclusion and Implications

Generally, the findings reveal that both conceptual and procedural knowledge are inseparable in student learning of mathematics. Mathematics teachers should thus promote student understanding by enhancing both knowledge types. Since in my study I investigated student's experiences, I would recommend further research to investigate teachers' view on the efficient sequencing. I also recommend that there is need for more research to explore if there are strategies teachers can use to determine which knowledge types to emphasize for certain topics.

References

Adu-Gyamfi, K., Bosse, M. J., & Faulconer, J. (2010). Assessing Understanding through Reading and Writing in Mathematics. *International Journal for Mathematics Teaching and Learning, 1–22.* Retrieved from http://www.cimt.plymouth.ac.uk/journal%5Cnfiles/148/adugyamfi.pdf

Bogdan, R. & Biklen, S. (2007). Qualitative Research for education: *An introduction to theories and method (5th Ed.)* Boston, MA: Allyn and Bacon.

Broody, A. J. (2007). An Alternative Reconceptualization of Procedural and Conceptual Knowledge Author (s): Arthur J. Baroody, Yingying Feil and Amanda R. Johnson Published by: National Council of Teachers of Mathematics.

Creswell, J. W. (2009). *Research design: Qualitative, quantitative and mixed method* *approaches (3rd ed.).* Thousand Oaks, CA: Sage Publications.

Denzin, N. K., & Lincoln, Y.S. (2005). The discipline and practice of qualitative research. In N.K. Denzin & Y.S. Lincoln (Eds.) *The Sage Handbook of Qualitative Research, 3rd edition.* Thousand Oaks, CA: Sage.

Fontana, A. & Frey, J. H. (2005). The interview: From neutral stance to political involvement. *In N. Denzin and Y. Lincoln (Eds.) Collecting and Interpreting Qualitative Materials (3rd ed.) (pp. 115-160). Los Angeles: Sage Publications.*

Hiebert, J. (Ed.). (2013). Conceptual and procedural knowledge: The case of mathematics. Routledge. National Council of Teachers of Mathematics (NCTM) (1995). *Assessment standards for school mathematics,* Reston, VA: Author

Porter, M. K., & Masingila, J. O. (2000). Examining the effects of writing on conceptual and procedural knowledge in calculus. *Educational Studies in Mathematics, 42(2), 165-177.*

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a One-Way Street: Bidirectional Relations Between Procedural and Conceptual Knowledge of Mathematics. *Educational Psychology Review, 27(4), 587–597*. Retrieved from https://doi.org/10.1007/s10648-015-9302-x

Rittle-Johnson, B., Wagner Alibali, M., Haverty, L., Heffernan, N., Koedinger, K., Nhouy vanis-vong, A., Rittle-Johnson or Martha Wagner Alibali, B. (1999). Conceptual and Procedural Knowledge of Mathematics: Does One Lead to the Other? *Journal of Educational Psychology*, *91*(1), 175–189. Retrieved from https://doi.org/10.1037/0022-0663.91.1.175

Schneider, M., & Stern, E. (2010). The developmental relations between conceptual and procedural knowledge: a multimethod approach. *Developmental Psychology, 46,178–192. doi:10.1037/a0016701*

Star, J. R. (2007). Foregrounding procedural knowledge. *Journal for Research in Mathematics Education, 132-135.*

Star, J. R. (2005). Reconceptualizing procedural knowledge. *Journal for Research in Mathematics Education, 36(5), 404-411.*

Xie, Y., Sharma, P., & Association for Educational Communication and Technology, W. D. (2005). Students lived experiences of using weblogs in a class: An exploratory study.